QCD corrections to the neutralino decay to an antisbottom and a bottom quark within MSSM

HANA HLUCHA

supervisor: Mgr. Karol Kovarik, Phd

work submited to: COMENIUS UNIVERSITY IN BRATISLAVA
FACULTY OF MATHEMATICS, PHYSICS AND INFORMATICS

Outline

- derivation of the lagrangian of the MSSM theory
  - ◇ notation
  - ◇ supersymmetry algebra
  - ◇ formalism of superspace and superfields
    - ∗ derivation of the supersymmetric generators, chiral and vector superfields, field strength superfields (abelian and non-abelian case)
  - ◇ lagrangian

- calculation of the neutralino decay
  - ◇ particle spectrum of the MSSM, superpotential, $\mathcal{L}_{\text{soft}}$ - lagrangian
  - ◇ couplings - relevant to the process
  - ◇ renormalization of the MSSM (sfermions, fermions)
  - ◇ QCD corrections: vertex, wave function and counterterm corrections
  - ◇ by-hand calculation of generic diagrams
  - ◇ use of Mathematica and LoopTools programs
  - ◇ graphs

- We use the metric $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$
- The Pauli matrices and the matrix σ^0 are defined as (in Peskin-Schroeder)

$$\sigma^0 := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \sigma^1 := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^2 := \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^3 := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- In the Weyl representation, the Dirac matrices γ^μ are given by

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

where $\sigma^\mu := (\sigma^0, \sigma^i)$, and $\bar{\sigma}^\mu := (\sigma^0, -\sigma^i)$

- Dirac bispinor: $\psi = \begin{pmatrix} \psi_L \\ \bar{\psi}_R \end{pmatrix}$

The two-component objects ψ_L and $\bar{\psi}_R$ are called left-handed and right-handed Weyl spinors. Their transformation laws under rotations $\vec{\alpha}$ and boosts $\vec{\beta}$ are

$$\begin{array}{lllcl} \psi_L \to A\psi_L & \text{where} & A & = & \exp(-\frac{i}{2}\vec{\alpha}\vec{\sigma} - \frac{1}{2}\vec{\beta}\vec{\sigma}) \\ \psi_R \to (A^{-1})^+\psi_R & \text{where} & (A^+)^{-1} & = & \exp(-\frac{i}{2}\vec{\alpha}\vec{\sigma} + \frac{1}{2}\vec{\beta}\vec{\sigma}) \end{array}$$

Notation, Weyl spinors

- two inequivalent spinor representations of SL(2,C)

  1. self-representation: $\chi_a \to A_a^{\ b}\chi_b$
  2. complex conjugate self-representation: $\bar{\eta}_{\dot{a}} \to A^{*\ \dot{b}}_{\dot{a}}\bar{\eta}_{\dot{b}} \Leftrightarrow \bar{\eta}^{\dot{a}}(A^{-1})^{+\ \dot{a}}_{\ \dot{b}}\bar{\eta}^{\dot{b}}$

- summation convention: $\chi\eta = \chi^a\eta_a, \quad \bar{\chi}\bar{\eta} = \bar{\chi}_{\dot{d}}\bar{\eta}^{\dot{d}}$

- structure of Dirac spinor: $\Psi \leftrightarrow \begin{pmatrix} \Psi_{L\ a} \\ \bar{\Psi}_R^{\ \dot{d}} \end{pmatrix}$ (from trafos under boosts & rotations)

- two dimensional antisymmetric metric tensor $\varepsilon \to$ rising, lowering indices $(\varepsilon^{12} = 1)$

- index structure of sigma matrices: $\sigma^\mu_{\ a\dot{d}}, \quad \bar{\sigma}^{\mu\ \dot{d}a}$ $\quad (A\sigma^\mu A^\dagger = (\Lambda^{-1})^\mu_{\ \nu}\sigma^\nu)$

- definition of $\sigma^{\mu\nu}$, $\bar{\sigma}^{\mu\nu}$:

$$
\begin{aligned}
\sigma^{\mu\nu} &:= \tfrac{1}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu) \\
\bar{\sigma}^{\mu\nu} &:= \tfrac{1}{4}(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu)
\end{aligned}
$$

- supersymmetry is a modern theory, that provides (partially) answers on the questions: hierarchy problem, gauge coupling unification, electroweak symmetry breaking, dark matter
- theorem of Coleman and Mandula - the most general Lie algebra of symmetries of S-matrix: $P_\mu, M_{\mu\nu},$ and finite number of Lorentz scalar operators B_l $(e.g. SU(n))$
- Haag, Lopuszanski and Sohnius theorem - extension of the Poincaré algebra $\longrightarrow$ superalgebra, introduction of the fermionic generators Q, MSSM $\leftrightarrow$ the simplest superalgebra (one set of the generators Q)

$$
\begin{array}{rclrcl}
[P^\mu, P^\nu] & = & 0 & [M^{\mu\nu}, M^{\rho\sigma}] & = & i(\eta^{\nu\rho} M^{\mu\sigma} + \eta^{\mu\sigma} M^{\nu\rho} - \mu \leftrightarrow \nu) \\
[P^\mu, Q_a] & = & ? & [M^{\mu\nu}, P^\lambda] & = & i(\eta^{\nu\lambda} P^\mu - \eta^{\mu\lambda} P^\nu) \\
[P^\mu, \bar{Q}_{\dot{a}}] & = & ? & [M^{\mu\nu}, Q_a] & = & ? \\
\{Q_a, Q_b\} & = & ? & [M^{\mu\nu}, \bar{Q}^{\dot{a}}] & = & ? \\
\{\bar{Q}_{\dot{a}}, \bar{Q}_{\dot{b}}\} & = & ? & \{Q_a, \bar{Q}_{\dot{b}}\} & = & ?
\end{array}
$$

- MSSM $\leftrightarrow$ 2 Higgs doublets (minimal choice)

- $[P^\mu, Q_a] = 0$ since translations act only on the argument of a spinor field
- $\{Q_a, Q^b\} = s(\sigma^{\mu\nu})_a{}^b M_{\mu\nu}$, but $\{Q_a, Q^b\}$ commutes with $P_\mu \Rightarrow s = 0$
- since Q_a is a Weyl spinor its transformations with respect to the Lorentz group are already determined

$$\Lambda = e^{-\frac{i}{2}\omega_{\mu\nu}M^{\mu\nu}} \quad \text{and its representation} \quad S(\Lambda) = e^{-\frac{i}{2}\omega_{\mu\nu}\frac{1}{2}\Sigma^{\mu\nu}}$$

$$Q'_a = (1 + \tfrac{1}{2}\omega_{\mu\nu}\sigma^{\mu\nu})_a{}^b Q_b = U(\Lambda)^\dagger Q_a U(\Lambda) = Q_a + \tfrac{i}{2}\omega_{\mu\nu}[M^{\mu\nu}, Q_a] \Rightarrow$$

$$\Rightarrow \quad [M^{\mu\nu}, Q_a] = -i(\sigma^{\mu\nu})_a{}^b Q_b$$

- $\{Q_a, \bar{Q}_{\dot{b}}\} = t\sigma^\mu_{a\dot{b}} P_\mu$ no restriction on t, convention: $t = 2$

- superalgebra's (anti)commutation relations

$$
\begin{aligned}
[P^\mu, P^\nu] &= 0 & [M^{\mu\nu}, M^{\rho\sigma}] &= i(\eta^{\nu\rho}M^{\mu\sigma} + \eta^{\mu\sigma}M^{\nu\rho} - \mu \leftrightarrow \nu) \\
[P^\mu, Q_a] &= 0 & [M^{\mu\nu}, P^\lambda] &= i(\eta^{\nu\lambda}P^\mu - \eta^{\mu\lambda}P^\nu) \\
[P^\mu, \bar{Q}_{\dot{a}}] &= 0 & [M^{\mu\nu}, Q_a] &= -i(\sigma^{\mu\nu})_a{}^b Q_b \\
\{Q_a, Q_b\} &= 0 & [M^{\mu\nu}, \bar{Q}^{\dot{a}}] &= -i(\bar{\sigma}^{\mu\nu})^{\dot{a}}{}_{\dot{b}} Q^{\dot{b}} \\
\{\bar{Q}_{\dot{a}}, \bar{Q}_{\dot{b}}\} &= 0 & \{Q_a, \bar{Q}_{\dot{b}}\} &= 2\sigma^\mu_{a\dot{b}} P_\mu
\end{aligned}
$$

- internal symmetry group

$$[B_i, B_j] = c_{ij}^k B_k \qquad SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

- irreducible representations
  - ◇ squared momentum P^2 is a Casimir operator
  - ◇ the squared of the Pauli-Ljubanski vector $W^\mu = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}M_{\nu\rho}P_\sigma$ is not a Casimir operator, $[W^2, Q] \neq 0$

- construction of an invariant operator

  1. define a pseudovector $X_\mu := \frac{1}{2}\bar{Q}\gamma_\mu\gamma_5 Q$
  2. define a new vector $B_\mu := W_\mu + \frac{1}{4}X_\mu$
  3. then define a tensor $C_{\mu\nu} := B_\mu P_\nu - B_\nu P_\mu$

  then $C^2 = C_{\mu\nu}C^{\mu\nu}$ is a Casimir operator

- irreps are characterized by eigenvalues of P^2 and C^2; $m^2, j(j+1)$

rest frame

		$s_3 = j_3$	$1\,\lvert\Omega\rangle$
	$j_3 = j$	$s_3 = j_3 + \frac{1}{2}$	$\bar{Q}^{\dot{1}}\,\lvert\Omega\rangle$
	$j_3 = j + \frac{1}{2}$	$s_3 = j_3 - \frac{1}{2}$	$\bar{Q}^{\dot{2}}\,\lvert\Omega\rangle$
(m, j)	$\vdots$	$s_3 = j_3$	$\bar{Q}^{\dot{1}}\bar{Q}^{\dot{2}}\,\lvert\Omega\rangle$
	$j_3 = -j$	$\uparrow$	
		spin eigenstates	

$\lvert\Omega\rangle$ - Clifford vacuum

$Q_a\,\lvert\Omega\rangle = 0$ (fixed j_3)

$C^2 = 2m^4 J_k J^k$

$[J_k, Q_a] = 0$

Further properties

- an irrep has equal number of bosonic and fermionic states

  proof: $Q_a(-1)^{N_F}|\,\rangle = (-1)^{N_F-1}Q_a|\,\rangle \quad \longrightarrow \quad 0 = \mathrm{Tr}\,[(-1)^{N_F}\{Q_a,\bar{Q}_{\dot{b}}\}]$

  $\{Q_a,\bar{Q}_{\dot{b}}\} = 2\sigma^{\mu}_{a\dot{b}}P_\mu \quad \longrightarrow \quad 0 = 2\sigma^{\mu}_{a\dot{b}}P_\mu \mathrm{Tr}[(-1)^{N_F}]$

  $$\Downarrow$$

  $$0 = \mathrm{Tr}[(-1)^{N_F}]$$

- the particles and its superpartners poses equal masses

  proof: it follows from the zero commutator $[P^2, Q]$
  not observed $\longrightarrow$ broken supersymmetry

Superspace and superfields

- superspace coordinates: $(x^\mu, \theta_a, \bar\theta_{\dot a})$, $\quad a = 1,2 \quad \dot a = \dot 1, \dot 2$
- superfield: function on the superspace; expansion in the parameters θ a $\bar\theta$:

$$\begin{aligned}\Phi(x,\theta,\bar\theta) &= f(x) + \theta^a \phi_a(x) + \bar\theta_{\dot a}\bar\chi^{\dot a}(x) + (\theta\theta)m(x) + (\bar\theta\bar\theta)n(x) \\ &+ (\theta\sigma^\mu\bar\theta)V_\mu(x) + (\theta\theta)\bar\theta_{\dot a}\bar\lambda^{\dot a}(x) + (\bar\theta\bar\theta)\theta^a\psi_a(x) + (\theta\theta)(\bar\theta\bar\theta)d(x)\end{aligned}$$

- element of the supergroup:

$$G(x,\theta,\bar\theta) = \exp[i(-x^\mu P_\mu + \theta Q + \bar\theta\bar Q)]$$

- product of the group elements induce a motion in the parametric space $(x,\theta,\bar\theta)$: $G(x,\theta,\bar\theta)G(a,\xi,\bar\xi) = G(B)$: $(x,\theta,\bar\theta) \to B$ (right action)

$$B = x^\mu + a^\mu + i(\xi\sigma^\mu\bar\theta) - i(\theta\sigma^\mu\bar\xi), \theta + \xi, \bar\theta + \bar\xi$$

- action of the Susy algebra on a superfield is generated by $P, Q, \bar{Q}$

$$
\begin{aligned}
\Phi(B) &= \Phi(x,\theta,\bar{\theta}) + (a^\mu + i\xi\sigma^\mu\bar{\theta} - i\theta\sigma^\mu\bar{\xi})\frac{\partial\Phi}{\partial x^\mu} + \xi^a\frac{\partial\Phi}{\partial\theta^a} + \xi_{\dot{a}}\frac{\partial\Phi}{\partial\theta_{\dot{a}}} + \ldots \\
&\stackrel{!}{=} \Big(1 - ia^\mu P_\mu + i\xi Q + i\bar{\xi}\bar{Q} + \ldots\Big)\Phi(x,\theta,\bar{\theta})
\end{aligned}
$$

- linear representation of the Susy algebra in terms of diff. operators

$$
\begin{aligned}
P_\mu &= i\partial_\mu \\
iQ_a &= \frac{\partial}{\partial\theta^a} + i(\sigma^\mu)_{a\dot{b}}\bar{\theta}^{\dot{b}}\partial_\mu \\
i\bar{Q}^{\dot{a}} &= \frac{\partial}{\partial\bar{\theta}_{\dot{a}}} + i(\bar{\sigma}^\mu)^{\dot{a}b}\theta_b\partial_\mu
\end{aligned}
$$

- the commutator relations for $P, Q, \bar{Q}$ are fulfilled
- sypersymmetric transformation: $\Phi \to \Phi + \delta_S\Phi; \quad \delta_S = i(\xi Q + \bar{\xi}\bar{Q})$

Superspace and superfields

- general superfield does not provide an irreducible representation of Susy algebra $\longrightarrow$ further constraints on superfield that are covariant under Susy algebra
- these constraints provide covariant derivatives which commute with δ_S

$$
\begin{aligned}
D_a &= \partial_a - i(\sigma^\mu)_{a\dot{b}}\bar{\theta}^{\dot{b}}\partial_\mu \\
\bar{D}^{\dot{a}} &= \bar{\partial}^{\dot{a}} - i(\bar{\sigma}^\mu)^{\dot{a}b}\theta_b\partial_\mu
\end{aligned}
$$

- we differentiate three types of superfields:
  - ◇ left-handed chiral superfields: $\bar{D}_{\dot{a}}\Phi = 0$
  - ◇ right-handed chiral superfields: $D_a\Phi^\dagger = 0$
  - ◇ vector supefields: $\Phi = \Phi^\dagger$

- Left-handed chiral superfield ($\bar{D}_{\dot{a}}\Phi = 0$)
  - ◇ it is convenient to switch to new variables: $(x, \theta', \bar{\theta}') \longrightarrow (y, \theta, \bar{\theta})$ where $y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}, \theta'_a = \theta_a, \theta'_{\dot{a}} = \theta_{\dot{a}}$
  - ◇ the covariant derivative becomes simple: $\bar{D}_{\dot{a}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{a}}}\Big|_{y,\theta}$
  - ◇ in these new variables: $\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) + (\theta\theta)F(y)$
- Right-handed chiral superfield ($D_a\Phi^\dagger = 0$)
  - ◇ $z^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$: $\Phi^\dagger(z,\bar{\theta}) = \phi^*(z) + \sqrt{2}\bar{\theta}\bar{\psi}(z) + (\bar{\theta}\bar{\theta})F^*(z)$
- product of left-(right-) handed superfields is again left-(right-)handed superfield
- susy-transformation of component fields

$$
\begin{aligned}
\delta_S\phi &= \sqrt{2}\xi\psi \\
\delta_S\psi_a &= \sqrt{2}\xi_a F - i\sqrt{2}\sigma^\mu{}_{a\dot{b}}\bar{\xi}^{\dot{b}}\partial_\mu\phi \\
\triangleright\quad \delta_S F &= i\sqrt{2}\partial_\mu(\psi\sigma^\mu\bar{\xi})
\end{aligned}
$$

- Vector superfield ($\Phi = \Phi^\dagger$)

$$\begin{aligned} V(x,\theta,\bar\theta) &= C(x) + \theta\chi(x) + \bar\theta\bar\chi(x) + (\theta\theta)M(x) + (\bar\theta\bar\theta)M^*(x) \\ &+ \theta\sigma^\mu\bar\theta V_\mu(x) + (\theta\theta)\bar\theta[\bar\lambda(x) - \frac{i}{2}\bar\sigma^\mu\partial_\mu\chi(x)] \\ &+ (\bar\theta\bar\theta)\theta[\lambda(x) - \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)] + (\theta\theta)(\bar\theta\bar\theta)[\frac{1}{2}D(x) - \frac{1}{4}\partial_\mu\partial^\mu C(x)] \end{aligned}$$

- gauge transformation: $V(x,\theta,\bar\theta) \to V(x,\theta,\bar\theta) + \Phi(x,\theta,\bar\theta) + \Phi^\dagger(x,\theta,\bar\theta)$

$$(\Phi^\dagger + \Phi) = i\theta\sigma^\mu\bar\theta\partial_\mu(\phi^* - \phi) + \dots$$

- Wess-Zumino gauge: fields C, χ, M are gauged away; λ, D are invariant, imaginary part of ϕ is not fixed

$$V_{WZ}(x,\theta,\bar\theta) = \theta\sigma^\mu\bar\theta V_\mu(x) + i(\theta\theta)\bar\theta\bar\lambda(x) - i(\bar\theta\bar\theta)\theta\lambda(x) + \tfrac{1}{2}(\theta\theta)(\bar\theta\bar\theta)D(x)$$

- Field strength superfields

$$\begin{aligned} W_a &= -\frac{1}{4}(\bar{D}\bar{D})D_a V \\ \bar{W}_{\dot{a}} &= -\frac{1}{4}(DD)\bar{D}_{\dot{a}} V \end{aligned}$$

- component expansion

$$\begin{aligned} W_a(y) &= -i\lambda_a(y) + \theta_a D(y) - (\sigma^{\mu\nu}\theta)_a V_{\mu\nu}(y) - (\theta\theta)(\sigma^\mu \partial_\mu \bar{\lambda}(y))_a \\ \bar{W}_{\dot{a}}(z) &= +i\bar{\lambda}_{\dot{a}}(z) + \bar{\theta}_{\dot{a}} D(z) + \varepsilon_{\dot{a}\dot{b}}(\bar{\sigma}^{\mu\nu}\bar{\theta})^{\dot{b}} V_{\mu\nu}(z) - (\bar{\theta}\bar{\theta})(\partial_\mu \lambda(z)\sigma^\mu)_{\dot{a}} \end{aligned}$$

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$$

- lagrangian (abelian case): $\mathcal{L} = \mathcal{L}_\Phi + \mathcal{L}_W$

$$\begin{aligned} \mathcal{L}_\Phi &= \Phi_i^\dagger \Phi_i\Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \left[\left(\frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}b_{ijk}\Phi_i\Phi_j\Phi_k + c_i\Phi_i\right)\Big|_{\theta\theta} + \text{h.c.}\right] \\ \mathcal{L}_W &= \frac{1}{4}\left(W^a W_a|_{\theta\theta} + \bar{W}_{\dot{a}}\bar{W}^{\dot{a}}|_{\bar{\theta}\bar{\theta}}\right) \end{aligned}$$

- supersymmetric local gauge invariant lagrangian:

$$\mathcal{L} = \frac{1}{16kg^2}\mathrm{Tr}\left(W^a W_a\Big|_{\theta\theta} + \bar{W}_{\dot{a}}\bar{W}^{\dot{a}}\Big|_{\bar{\theta}\bar{\theta}}\right) + \Phi^\dagger e^{2gV}\Phi\Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \left(W + \mathrm{h.c.}\right)$$

- local gauge:

$$\Phi' = e^{-i2g\Lambda(x)}\Phi, \qquad e^{2gV'(x)} = e^{-i2g\Lambda^\dagger(x)}e^{2gV(x)}e^{i2g\Lambda(x)}$$
$$\Lambda_{ij} = T^{(a)}_{ij}\Lambda^{(a)}, \qquad V_{ij} = V^{(a)}T^{(a)}_{ij}$$

- field strength superfields:

$$\begin{aligned} W_a &= -\frac{1}{4}\bar{D}\bar{D}e^{-2gV}D_a e^{2gV} \\ \bar{W}_{\dot{a}} &= -\frac{1}{4}DD\left(\bar{D}_{\dot{a}}e^{2gV}\right)e^{-2gV} \end{aligned}$$

Lagrangian in components

$$
\begin{aligned}
\mathcal{L} &= i\lambda^{(a)}\sigma^{\mu}\mathscr{D}_{\mu}\bar{\lambda}^{(a)} - \frac{1}{4}F^{(a)}_{\mu\nu}F^{(a)\ \mu\nu} + \frac{1}{2}D^{(a)}D^{(a)} \\
&+ i(\bar{\psi}_i\bar{\sigma}^{\mu}\mathscr{D}_{\mu}\psi_i) + F_i^*F_i + (\mathscr{D}_{\mu}\phi_i)^*(\mathscr{D}^{\mu}\phi_i) \\
&+ i\sqrt{2}gT^{(a)}_{ij}[\phi_i^*(\lambda^{(a)}\psi_j) - (\bar{\lambda}^{(a)}\bar{\psi}_i)\phi_j] + gD^{(a)}T^{(a)}_{ij}\phi_i^*\phi_j
\end{aligned}
$$

where the covariant derivatives and the field strength tensor are:

$$
\begin{aligned}
\mathscr{D}_{\mu}\bar{\lambda}^{(a)} &= \partial_{\mu}\bar{\lambda}^{(a)} - gf^{abc}V^{(b)}_{\mu}\bar{\lambda}^{(c)} \\
F^{(a)}_{\mu\nu} &= \partial_{\mu}V^{(a)}_{\nu} - \partial_{\nu}V^{(a)}_{\mu} - gf^{abc}V^{(b)}_{\mu}V^{(c)}_{\nu} \\
\mathscr{D}_{\mu}\psi &= \partial_{\mu}\psi + igV_{\mu}\psi \\
\mathscr{D}_{\mu}\phi &= \partial_{\mu}\phi + igV_{\mu}\phi
\end{aligned}
$$

Particle content

Superfield	Particle	Spin	G	Superpartner	Spin
$\hat{V}_1$	B_μ	1	(1, 1, 0)	$\tilde{B}$	$\frac{1}{2}$
$\hat{V}_2$	W^i_μ	1	(1, 3, 0)	$\tilde{W}^i$	$\frac{1}{2}$
$\hat{V}_3$	G^a_μ	1	(8, 1, 0)	$\tilde{g}^a$	$\frac{1}{2}$
$\hat{Q}$	$Q = (u_L, d_L)$	$\frac{1}{2}$	$(3, 2, \frac{1}{3})$	$\tilde{Q} = (\tilde{u}_L, \tilde{d}_L)$	0
$\hat{U}^c$	$U^c = \bar{u}_R$	$\frac{1}{2}$	$(3^*, 1, \text{-}\frac{4}{3})$	$\tilde{U}^c = \tilde{u}^*_R$	0
$\hat{D}^c$	$D^c = \bar{d}_R$	$\frac{1}{2}$	$(3^*, 1, \frac{2}{3})$	$\tilde{D}^c = \tilde{d}^*_R$	0
$\hat{L}$	$L = (\nu_L, e_L)$	$\frac{1}{2}$	(1, 2, -1)	$\tilde{L} = (\tilde{\nu}_L, \tilde{e}_L)$	0
$\hat{E}^c$	$E^c = \bar{e}_R$	$\frac{1}{2}$	(1, 1, 2)	$\tilde{E}^c = \tilde{e}^*_R$	0
$\hat{H}_1$	$H_1 = (H^0_1, H^-_1)$	0	(1, 2, -1)	$\tilde{H}_1 = (\tilde{H}^0_1, \tilde{H}^-_1)$	$\frac{1}{2}$
$\hat{H}_2$	$H_2 = (H^+_2, H^0_2)$	0	(1, 2, 1)	$\tilde{H}_2 = (\tilde{H}^+_2, \tilde{H}^0_2)$	$\frac{1}{2}$

- Superpotential W

$$W = -\varepsilon_{ij}\Big[h_e \hat{H}_1^i \hat{L}^j \hat{E}^c + h_d \hat{H}_1^i \hat{Q}^j \hat{D}^c + h_u \hat{H}_2^j \hat{Q}^i \hat{U}^c - \mu \hat{H}_1^i \hat{H}_2^j\Big] + \text{h.c.}$$

- Soft susy-breaking lagrangian

$$\begin{aligned}
-\mathcal{L}_{\text{soft}} &= m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 - m_{12}^2 \varepsilon_{ij} (H_1^i H_2^j + H_1^{\dagger i} H_2^{\dagger j}) \\
&+ \frac{1}{2} m_{\tilde{g}} \tilde{g}^a \tilde{g}^a + \frac{1}{2} M \tilde{W}^i \tilde{W}^i + \frac{1}{2} M' \tilde{B} \tilde{B} \\
&+ M_{\tilde{Q}}^2 |\tilde{q}_L|^2 + M_{\tilde{U}}^2 |\tilde{u}_R^c|^2 + M_{\tilde{D}}^2 |\tilde{d}_R^c|^2 + M_{\tilde{L}}^2 |\tilde{l}_L|^2 + M_{\tilde{E}}^2 |\tilde{e}_R^c|^2 \\
&- \varepsilon_{ij} \Big(h_e A_e H_1^i \tilde{L}^j \tilde{E}^c + h_d A_d H_1^i \tilde{Q}^j \tilde{D}^c + h_u A_u H_2^j \tilde{Q}^i \tilde{U}^c + \text{h.c.} \Big)
\end{aligned}$$

- having all these, one can analyze Higgs sector, mass matrices, particle mass eigenstates, derive vertices, . . .

Neutralino

- raises up as a combination of $(\tilde{B}, \tilde{W}_3^0, \tilde{H}_1^0, \tilde{H}_2^0) \leftrightarrow \psi^0$
- mass lagrangian
$$\mathcal{L} = -\tfrac{1}{2}(\psi^0)^T Y \psi^0 + \mathrm{h.c.}$$
- with neutralino mass matrix
$$Y = \begin{pmatrix} M' & 0 & -m_Z s_W \cos\beta & m_Z s_W \sin\beta \\ 0 & M & m_Z c_W \cos\beta & -m_Z c_W \sin\beta \\ -m_Z s_W \cos\beta & m_Z c_W \cos\beta & 0 & -\mu \\ m_Z s_W \sin\beta & -m_Z c_W \sin\beta & -\mu & 0 \end{pmatrix}$$
- diagonalization of the mass matrix: $ZYZ^{-1} \rightarrow D$ (real Z, neg. eigenv. allowed)
- neutralino fields
$$\tilde{\chi}_i^0 \equiv Z_{ij} \begin{pmatrix} \psi_j^0 \\ \bar{\psi}_j^0 \end{pmatrix} \qquad (i = 1, 2, 3, 4)$$
- neutralino is a Majorana spinor

Sfermion

- sfermion mass matrix

$$\mathcal{M}^2_{\tilde{f}} = \begin{pmatrix} m^2_{\tilde{f}_L} & a_f m_f \\ a_f m_f & m^2_{\tilde{f}_R} \end{pmatrix}$$

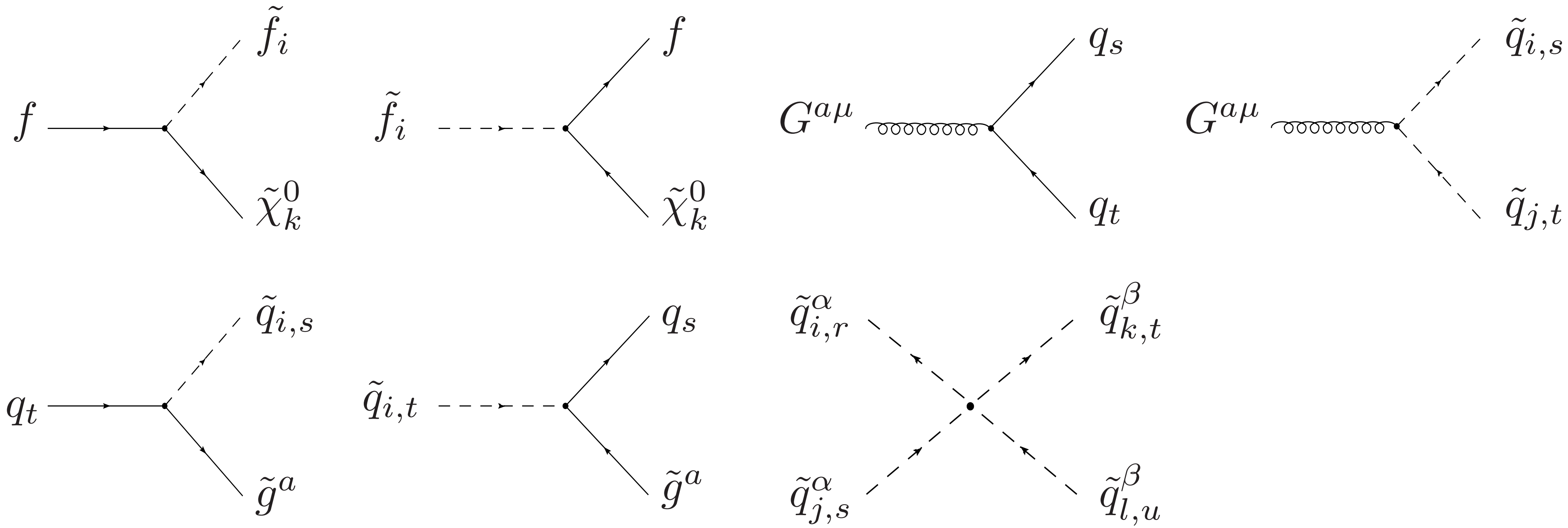
$$\begin{aligned} m^2_{\tilde{f}_L} &= M^2_{\{\tilde{Q},\tilde{L}\}} + (I_f^{3L} - e_f s_W^2)\cos 2\beta m_Z^2 + m_f^2 \\ m^2_{\tilde{f}_R} &= M^2_{\{\tilde{U},\tilde{D},\tilde{E}\}} + e_f s_W^2 \cos 2\beta m_Z^2 + m_f^2 \\ a_f &= A_f - \mu(\tan\beta)^{-2I_f^{3L}} \end{aligned}$$

- diagonalization - introducing mixing angle

$$\mathcal{M}^2_{\tilde{f}} = \begin{pmatrix} m^2_{\tilde{f}_L} & a_f m_f \\ a_f m_f & m^2_{\tilde{f}_R} \end{pmatrix} = \left(R^{\tilde{f}}\right)^\dagger \begin{pmatrix} m^2_{\tilde{f}_1} & 0 \\ 0 & m^2_{\tilde{f}_2} \end{pmatrix} \left(R^{\tilde{f}}\right)$$

where the mixing matrix is $\left(R^{\tilde{f}}\right) = \begin{pmatrix} \cos\theta_{\tilde{f}} & \sin\theta_{\tilde{f}} \\ -\sin\theta_{\tilde{f}} & \cos\theta_{\tilde{f}} \end{pmatrix}$

- important couplings for my calculation
  - neutralino - fermion - sfermion
  - gluon - fermion - fermion
  - gluon - sfermion - sfermion
  - gluino - fermion - sfermion
  - 4 sfermions



Neutralino-Sfermion-Fermion coupling

- the whole lagrangian reads

$$\mathcal{L} = \bar{f}\Big(a_{ik}^{\tilde{f}} P_R + b_{ik}^{\tilde{f}} P_L\Big)\tilde{\chi}_k^0 \tilde{f}_i + \bar{\tilde{\chi}}_k^0\Big(a_{ik}^{\tilde{f}} P_L + b_{ik}^{\tilde{f}} P_R\Big) f \tilde{f}_i^*$$

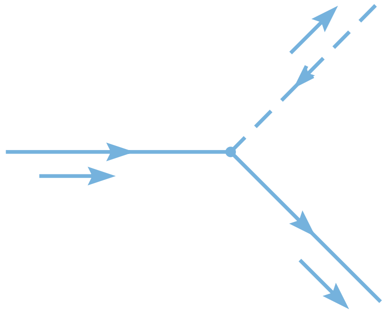
where

$$\begin{aligned} a_{ik}^{\tilde{f}} &= h_f Z_{kx} R_{i2}^{\tilde{f}} + g f_{Lk}^{f} R_{i1}^{\tilde{f}} \\ b_{ik}^{\tilde{f}} &= h_f Z_{kx} R_{i1}^{\tilde{f}} + g f_{Rk}^{f} R_{i2}^{\tilde{f}} \end{aligned}$$

$$\begin{aligned} f_{Lk}^{f} &= \sqrt{2}((e_f - I_f^{3L})\tan\theta_{\mathrm{W}} Z_{k1} + I_f^{3L} Z_{k2}) \\ f_{Rk}^{f} &= -\sqrt{2} e_f \tan\theta_{\mathrm{W}} Z_{k1} \end{aligned}$$

where x takes the values {3, 4} for {down, up} - type case, respectively

- Feynman's diagram for the neutralino decay to fermion and antisfermion: $(\tilde{\chi}^0_k \to \tilde{f}_i + f) \leftrightarrow (p_3 \to p_1 + p_2)$



$$\mathcal{M}_0 = i\bar{u}(p_2)\Big(a^{\tilde{f}}_{ik} P_R + b^{\tilde{f}}_{ik} P_L\Big)u(p_3)$$

- decay width - unpolarized case (in CMS system):

$$\Gamma_0 = \frac{p_f}{8\pi m^2_{\tilde{\chi}^0_k}} |\overline{\mathcal{M}_0}|^2$$

$$|\overline{\mathcal{M}_0}|^2 = \frac{1}{2}\mathrm{Tr}\Big[(\not{p}_2 + m_f)(a^{\tilde{f}}_{ik} P_R + b^{\tilde{f}}_{ik} P_L)(\not{p}_3 + m_{\tilde{\chi}^0_k})(a^{*\tilde{f}}_{ik} P_L + b^{*\tilde{f}}_{ik} P_R)\Big]$$

$$= p_2 \cdot p_3 (|a^{\tilde{f}}_{ik}|^2 + |b^{\tilde{f}}_{ik}|^2) + m_f m_{\tilde{\chi}^0_k} (a^{\tilde{f}}_{ik} b^{*\tilde{f}}_{ik} + a^{*\tilde{f}}_{ik} b^{\tilde{f}}_{ik})$$

Renormalization

- multiplicative renormalization :

$$\begin{aligned} \psi_0 &= \sqrt{Z_\psi}\psi &= \sqrt{1+\delta Z_\psi}\psi \\ m_0 &= Z_m m &= m + \delta m \end{aligned}$$

- on - shell renormalization: m - physical mass (pole of propagator)

- renormalization condition for the wave function: residuum of the propagator by $p^2 = m^2$ equals one

- Feynman's rules: "old" Feyn. rules + "new" Feyn. rules for the counterterms

$$\mathcal{L} = \mathcal{L}|_{\psi_0 \to \psi} + \delta\mathcal{L}$$

Renormalization of fermions (without mixing)

- splitting of the bare parameters

$$\begin{aligned} f_0 &\to (1 + \frac{1}{2}\delta Z^L P_L + \frac{1}{2}\delta Z^R P_R) f \\ \bar{f}_0 &\to \bar{f}(1 + \frac{1}{2}\delta Z^{L\dagger} P_R + \frac{1}{2}\delta Z^{R\dagger} P_L) \\ m_0 &\to m + \delta m \end{aligned}$$

- mass renormalization condition $\widetilde{\mathrm{Re}}\hat{\Gamma}(p)u(p)\Big|_{p^2=m^2} = 0$ yields

$$\delta m = \tfrac{1}{2}\widetilde{\mathrm{Re}}\Big(m\Pi^L(m^2) + m\Pi^R(m^2) + \Pi^{S,L}(m^2) + \Pi^{S,R}(m^2)\Big)$$

- wave function renorm. condition $\lim_{p^2\to m^2} \frac{1}{\not{p} - m}\widetilde{\mathrm{Re}}\hat{\Gamma}(p)u(p) = u(p)$ yields

$$\begin{aligned} \delta Z^{L/R} &= -\Pi^{L/R}(m^2) + \tfrac{1}{2m}\Big(\Pi^{S,L/R}(m^2) - \Pi^{S,R/L}(m^2)\Big) \\ &- \tfrac{\partial}{\partial p^2}\Big[m^2\Big(\Pi^{L/R}(p^2) + \Pi^{R/L}(p^2)\Big) + m\Big(\Pi^{S,L/R}(p^2) + \Pi^{S,R/L}(p^2)\Big)\Big]\Big|_{p^2=m^2} \end{aligned}$$

Renormalization of sfermions

- splitting of the bare parameters

$$\tilde{f}_i \to (\delta_{ij} + \frac{1}{2}\delta Z_{ij})\tilde{f}_j$$
$$m_i^2 \to m_i^2 + \delta m_i^2$$

- renormalization conditions $\widetilde{\mathrm{Re}}\hat{\Gamma}_{ij}(p^2)\Big|_{p^2=m_j^2} = 0, \qquad \lim_{k^2\to m_i^2} \widetilde{\mathrm{Re}}\hat{\Gamma}_{ii}(p^2) = 1$

  yields:

$$\begin{aligned} \delta m_i^2 &= \widetilde{\mathrm{Re}}\Pi_{ii}(m_i^2) \\ \delta Z_{ij} &= \frac{2}{m_i^2 - m_j^2}\widetilde{\mathrm{Re}}\Pi_{ij}(m_j^2), \qquad i \neq j \\ \delta Z_{ii} &= -\widetilde{\mathrm{Re}}\frac{\partial}{\partial p^2}\Pi_{ii}(p^2)\Big|_{p^2=m_i^2} \end{aligned}$$

- mixing matrix counterterm is set to cancel the anti- hermitian part of the wave function correction

$$\delta R_{ij}^{\tilde{f}} = \sum_{k=1}^{2} \frac{1}{4}(\delta Z_{ik} - \delta Z_{ki}) R_{kj}^{\tilde{f}}$$

Mixing matrix (angle) renormalization

- two possible ways (Blank(diss.), Bartl et al.)
  - ⋄ renormalization after rotation
  - ⋄ renormalization before rotation

- renormalization after rotation

we start with the following lagrangian:

$$\mathcal{L} = \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}^+ k^2 \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix} - \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}^+ \mathcal{M}^2_{\tilde{f}} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}$$

which after rotation is

$$\mathcal{L} = \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}^+ k^2 \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} - \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}^+ \underbrace{U_{\tilde{f}}\, \mathcal{M}^2_{\tilde{f}}\, U^+_{\tilde{f}}}_{D_{\tilde{f}}} \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}$$

renormalization: $\tilde{f} \to \tilde{f} + \frac{1}{2}\delta Z, D_{\tilde{f}} \to D_{\tilde{f}} + \delta D_{\tilde{f}}$ leads to renorm $\mathcal{L}$ →self energies

from RCs we obtain $\delta m, \delta Z$

Mixing matrix (angle) renormalization

- now consider following interaction lagrangian

$$\mathcal{L} \sim C_\alpha(\chi f \tilde{f}_\alpha), \quad \alpha \epsilon \{L, R\}$$

field is first rotated to mass-eigenstate: $\tilde{f}_\alpha \to (U^+_{\tilde{f}})_{\alpha i} \tilde{f}_i$ and then renormalized

but we have to also renormalize mixing matrix: $U_{\tilde{f}} \to U_{\tilde{f}} + \delta U_{\tilde{f}}$

we obtain: $\mathcal{F}_{\chi f \tilde{f}} \to \tilde{C}_j(\delta_{ji} + \frac{1}{2}\delta Z^{\tilde{f}}_{ji} - (i\sigma_2)_{ji} \cdot \delta\theta_{\tilde{f}})$

- renormalization before rotation

at first, fields are renormalized: $\tilde{f}_{L/R} \to \tilde{f}_{L/R} + \frac{1}{2}\delta Z^{\tilde{f}}_{L/R}$

and then rotation is performed: $\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} \to U_{\tilde{f}} \begin{pmatrix} 1 + \frac{1}{2}\delta Z^{\tilde{f}}_L & 0 \\ 0 & 1 + \frac{1}{2}\delta Z^{\tilde{f}}_R \end{pmatrix} U^+_{\tilde{f}} \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}$

$$\to \begin{pmatrix} 1 + \frac{1}{2}\delta \tilde{Z}^{\tilde{f}}_L & 0 \\ 0 & 1 + \frac{1}{2}\delta \tilde{Z}^{\tilde{f}}_R \end{pmatrix} \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}$$

Mixing matrix (angle) renormalization

non-diagonal mass matrix: $U_{\tilde{f}}\delta\mathcal{M}^2_{\tilde{f}}U^+_{\tilde{f}} = (U_{\tilde{f}}\delta U^+_{\tilde{f}})\cdot d_{\tilde{f}} + \delta D_{\tilde{f}} + D_{\tilde{f}}\cdot(\delta U_{\tilde{f}}U^+_{\tilde{f}})$

we thus obtain renormalized $\mathcal{L}$ from which we can calculate renormalized self-energies

counterterms in this second method are denoted by tilde:

$$\theta_{\tilde{f}} \to \theta_{\tilde{f}} + \delta\tilde{\theta}_{\tilde{f}}, \quad m^2_{\tilde{f}} \to m^2_{\tilde{f}} + \delta\tilde{m}^2_{\tilde{f}}, \quad \delta\tilde{Z}^{\tilde{f}}_{ij}$$

analogously to first scheme: $\mathcal{F}_{\chi f\tilde{f}} \to \tilde{C}_j(\delta_{ji} + \frac{1}{2}\delta\tilde{Z}^{\tilde{f}}_{ji})$

counteterm to θ is absent because the renormalization was done before rotation

- comparison of both methods

the unrenormalized self-energies are scheme-independent therefore the divergent parts of renormalized self energies in both methods equal. This leads to following relation

$$\delta\tilde{\theta}_{\tilde{f}} = \frac{1}{4}(\delta Z^{\tilde{f}}_{12} - \delta Z^{\tilde{f}}_{21})$$

$\mathcal{F}_{\chi f\tilde{f}}$ also method independent: $\frac{1}{2}\delta Z^{\tilde{f}}_{ij} - (i\sigma_2)_{ij}\cdot\delta\theta_{\tilde{f}} = \frac{1}{2}\delta\tilde{Z}^{\tilde{f}}_{ij}$

we are left with equality of divergent parts of mixing angles: $\delta\theta_{\tilde{f}} \overset{\varepsilon}{=} \delta\tilde{\theta}_{\tilde{f}}$

- bare lagrangian

$$\mathcal{L}_0 = \bar{f}\Big(a^{\tilde{f}}_{ik}P_R + b^{\tilde{f}}_{ik}P_L\Big)\tilde{\chi}^0_k\tilde{f}_i \qquad \text{(fixed } i,k\text{)}$$
$$= \bar{f}_0\Big([h_f Z_{kx}R^{\tilde{f}}_{i2} + g f^f_{Lk}R^{\tilde{f}}_{i1}]P_R + [h_f Z_{kx}R^{\tilde{f}}_{i1} + g f^f_{Rk}R^{\tilde{f}}_{i2}]P_L\Big)\tilde{\chi}^0_k\tilde{f}_{i,0}$$

- under QCD corrections goes into

$$\mathcal{L} = \bar{f}(1 + \frac{1}{2}\delta Z^L P_R + \frac{1}{2}\delta Z^R P_L)\Big([(h_f + \delta h_f)Z_{kx}(R^{\tilde{f}}_{j2} + \delta R^{\tilde{f}}_{j2})$$
$$+ g f^f_{Lk}(R^{\tilde{f}}_{j1} + \delta R^{\tilde{f}}_{j1})]P_R\Big)\tilde{\chi}^0_k(\delta_{ji} + \frac{1}{2}\delta Z_{ji})\tilde{f}_i \quad + \ldots (b^{\tilde{f}}_{ik}P_L)$$

$$(v) \quad = \mathcal{L}_0(f,\tilde{f})$$
$$(w) \quad + \bar{f}\Big([\frac{1}{2}\delta Z^L\delta_{ji} + \frac{1}{2}\delta Z_{ji}]a^{\tilde{f}}_{jk}P_R\Big)\tilde{\chi}^0_k\tilde{f}_i \quad + \ldots (b^{\tilde{f}}_{jk}P_L)$$
$$(c) \quad + \bar{f}\Big(\frac{1}{m_f}h_f\delta m_f Z_{k3}R^{\tilde{f}}_{i2} + h_f Z_{k3}\delta R^{\tilde{f}}_{i2} + g f^f_{Lk}\delta R^{\tilde{f}}_{i1}\Big)P_R\tilde{\chi}^0_k\tilde{f}_i \quad + \ldots (P_L)$$

1-loop level

- amplitude at one loop level:

$$\mathcal{M}_1 = i\bar{u}(p_2)(AP_R + BP_L)u(p_3)$$

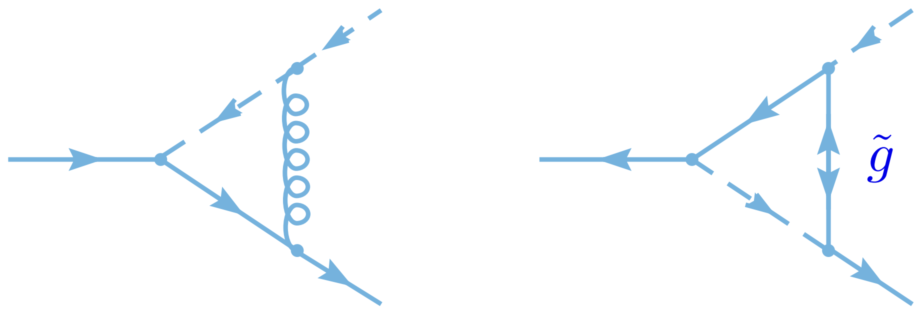
$$\begin{aligned} A &= a^{(v)} + a^{(w)} + a^{(c)} \\ B &= b^{(v)} + b^{(w)} + b^{(c)} \end{aligned}$$

  - ◇ (v) - vertex corrections
  - ◇ (w) - wave function corrections
  - ◇ (c) - counterterm corrections: $\delta h_f \leftrightarrow \delta m_f, \delta R_{ij}^{\tilde{f}}$

- "total" decay width:

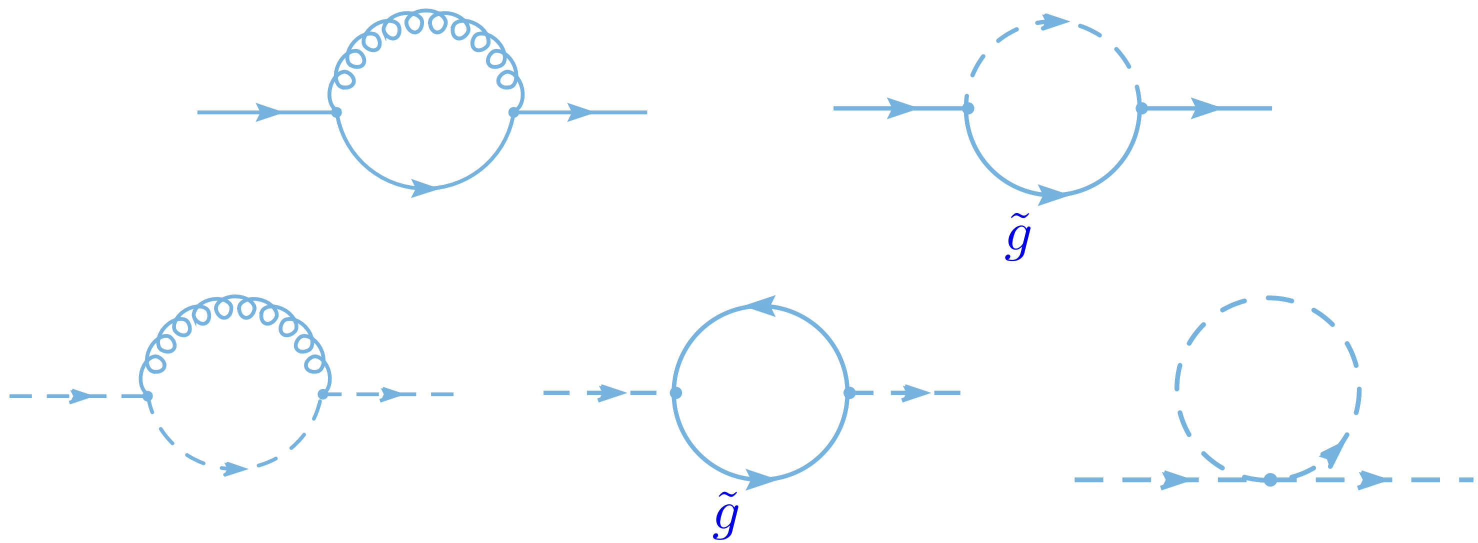
$$\Gamma = \frac{4\pi p_f}{32\pi^2 m_{\tilde{\chi}_k^0}^2} \left(C_F^0 |\overline{\mathcal{M}_0}|^2 + C_F^s \delta_s |\overline{\mathcal{M}_0}|^2 + C_F^1 2\mathrm{Re}[\mathcal{M}_0^* \mathcal{M}_1] \right)$$

- vertex corrections

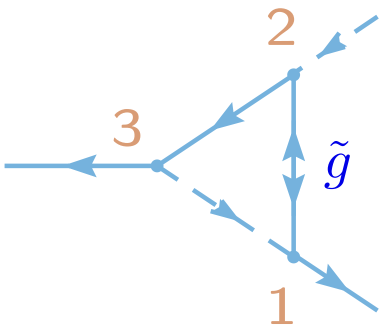


- special treatment

- fermion and sfermion self-energies



- vertex correction with gluino inside a loop



$$\ldots \bar{f}\Gamma_1\tilde{g}\tilde{f}\,|\,\bar{f}\Gamma_2\tilde{g}\tilde{f}\,|\,\bar{\tilde{\chi}}\Gamma_3 f\tilde{f}^* \ldots$$

$$\downarrow$$

$$\ldots \bar{f}\Gamma_1\tilde{g}\tilde{f}\,|\,\bar{\tilde{g}}^c\Gamma_2' f^c\tilde{f}\,|\,\bar{f}^c\Gamma_3'\tilde{\chi}^c\tilde{f}^* \ldots$$

discontinuous fermion number flow $\longrightarrow$ continuous fermion number flow

$\Gamma' = C\Gamma C^{-1} = \eta\Gamma$ $\eta = 1$ for $1, \gamma_5$ C is the charge conjugation operator

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\sum_s (a_p^s u^s(p)e^{-ipx} + b_p^{\dagger s} v^s(p)e^{ipx})$$

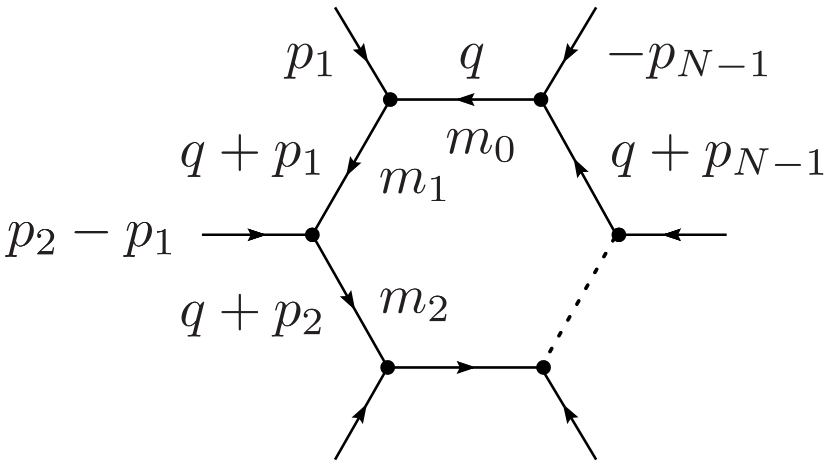
$$\psi^c(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\sum_s (a_p^{\dagger s} v^s(p)e^{ipx} + b_p^s u^s(p)e^{-ipx})$$

(A.Denner, H.Eck, O.Hahn, J.Kublbeck: Compact Feynman rules for Majorana fermions)

- general one loop integral

$$T^N_{\mu_1 \dots \mu_M}(p_1, \dots, p_{N-1}, m_0, \dots, m_{N-1}) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^D q \frac{q_{\mu_1} \dots q_{\mu_M}}{[q^2 - m_0^2 + i\varepsilon][(q+p_1)^2 - m_1^2 + i\varepsilon] \dots [(q+p_{N-1})^2 - m_{N-1}^2 + i\varepsilon]}$$

- convention for the momenta



$$T^1 \equiv A_0(m_0^2)$$
$$T^2 \equiv B_0(p_1^2, m_0^2, m_1^2)$$
$$T^3 \equiv C_0(p_1^2, (p_1-p_2)^2, p_2^2, m_0^2, m_1^2, m_2^2)$$

other tensor integrals $B^\mu, B^{\mu\nu}, C^\mu, C^{\mu\nu}$ etc. through tensor reduction procedure

$$B^{\mu\nu} = g^{\mu\nu} B_{00} + p_1^\mu p_1^\nu B_{11}, \dots$$

- UV divergent parts

Integral		UV divergent part
$A_0(m^2)$	$\longrightarrow$	$m^2\Delta$
B_0	$\longrightarrow$	Δ
B_1	$\longrightarrow$	$-\frac{1}{2}\Delta$
$B_{00}(k^2, m_0^2, m_1^2)$	$\longrightarrow$	$-\frac{1}{4}(k^2/3 - m_0^2 - m_1^2)\Delta$
B_{11}	$\longrightarrow$	$\frac{1}{3}\Delta$
C_{00}	$\longrightarrow$	$\frac{1}{4}\Delta$

- where $\Delta = \frac{1}{\varepsilon} - \gamma_E + \ln 4\pi$

- IR divergent parts

Integral		IR divergent part
$\dot{B}_0(m^2,\lambda^2,m^2) = \dot{B}_0(m^2,m^2,\lambda^2)$	$\longrightarrow$	$-\frac{\ln\lambda^2}{2m^2}$
$\dot{B}_1(m^2,m^2,\lambda^2)$	$\longrightarrow$	$\frac{\ln\lambda^2}{2m^2}$
$\dot{B}_1(m^2,\lambda^2,m^2)$	$\longrightarrow$	0
$\mathrm{Re}[C_0(m_1^2,m_0^2,m_2^2,\lambda^2,m_1^2,m_2^2)]$	$\longrightarrow$	$-\frac{\ln\beta_0}{\kappa}\ln\lambda^2$

- where

$$\kappa = \kappa(m_0^2,m_1^2,m_2^2) = \sqrt{\lambda(m_0^2,m_1^2,m_2^2)}$$

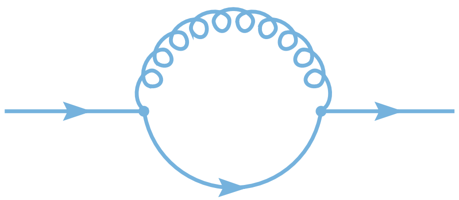
$$\beta_0 = \frac{m_0^2-m_1^2-m_2^2+\kappa}{2m_1m_2}$$

DREG vs. DRED

- in DREG, vector fields become D-dimensional $\longrightarrow$ contradiction with equality of fermionic and bosonic degrees of freedom $\longrightarrow$ need of a new regul. scheme, DRED
- in DREG, vector field has $D-2$ degrees of freedom, its superpartner has 2.
- missing degrees of freedom $(D = 4-\varepsilon)$ are taken into account through ε-scalar field

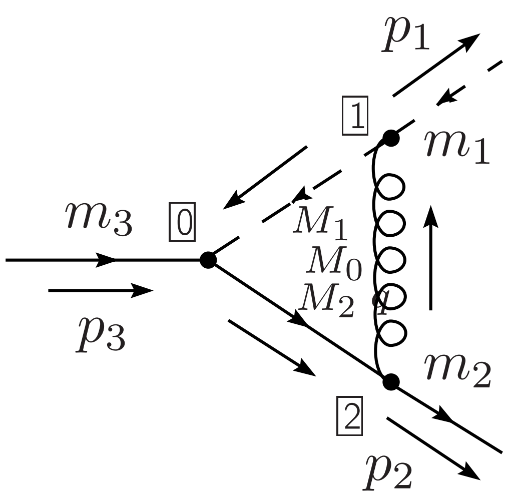
$$V_\mu = \begin{pmatrix} V_i \\ V_\sigma \end{pmatrix}, \; \gamma_\mu = \begin{pmatrix} \gamma_i \\ \gamma_\sigma \end{pmatrix}, \; p_\mu = \begin{pmatrix} p_i \\ 0 \end{pmatrix}, \text{ and } \mathcal{L}^4 = \mathcal{L}^D + \mathcal{L}^\varepsilon$$

- at one loop level, the difference is only in finite terms
- example



$$\Pi^{\overline{DR}}(p^2) = \tfrac{1}{4\pi^2} g_s^2 C_F [2\not{p} B_1 + 2(\not{p} - 2m_q) B_0]$$
$$\Pi^{DReg}(p^2) = \tfrac{1}{4\pi^2} g_s^2 C_F [2\not{p} B_1 + 2(\not{p} - 2m_q)(B_0 - \frac{1}{2})]$$

- vertex corrections



$$\mathcal{M} = \frac{i}{4\pi^2}\bar{u}(p_2)(A_R P_R + A_L P_L)u(p_3)$$

$\boxed{0}$: $i(g_0^L P_L + g_0^R P_R)$

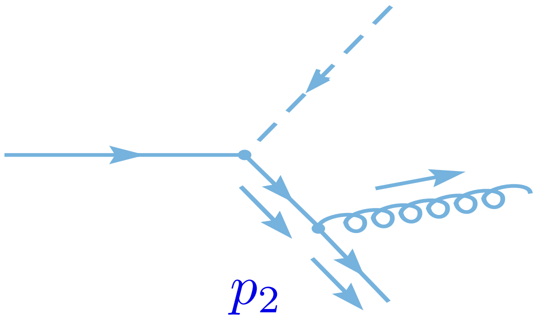
$\boxed{1}$: $ig_1(q - 2p_1)^\mu$

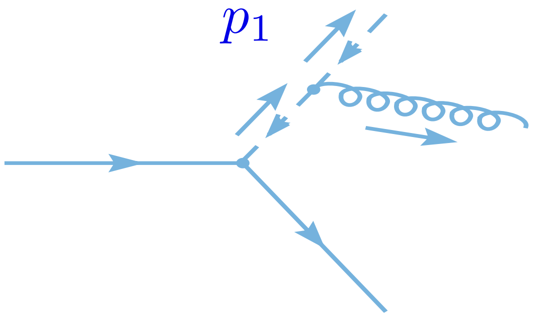
$\boxed{2}$: $i\gamma^\nu(g_2^L P_L + g_2^R P_R)$

$$\begin{aligned} {A_L}^{FSV} &= g_0^L g_1 g_2^R [2C_0(m_1^2 - m_3^2) + C_2(2m_1^2 + m_2^2 - 2m_3^2) + C_1(3m_1^2 - m_3^2) \\ &+ 4C_{00} + C_{11}m_1^2 + C_{12}(m_1^2 + m_2^2 - m_3^2) + C_{22}m_2^2] \\ &+ g_0^R g_1 g_2^L m_2 m_3 (2C_0 + 2C_2 + C_1) + g_0^L g_1 g_2^L m_2 (2M_2 C_0 + M_2 C_1 + M_2 C_2) \\ &+ m_3 g_0^R g_1 g_2^R (-2M_2 C_0 - M_2 C_1) \\ {A_R}^{FSV} &= A_L^{FSV}(M_0, M_1, M_2, g_0^L, g_0^R, g_1, g_2^L, g_2^R)(L \leftrightarrow R) \end{aligned}$$

- $a^{(v)} = \frac{1}{4\pi^2} {A_R}^{f\tilde{f}G}(\lambda, m_{\tilde{f}_i}, m_f, b_{ik}^{\tilde{f}}, a_{ik}^{\tilde{f}}, -g_s, -g_s, -g_s) + \frac{1}{4\pi^2} A_R^{\tilde{g}f\tilde{f}}(\ )$

- massles gluon in a loop $\longrightarrow$ IR - divergence





- soft gluon radiation:

$$\left(\frac{d\Gamma}{d\Omega}\right)_{\text{soft}} = \left(\frac{d\Gamma}{d\Omega}\right)_0 \delta_s$$

$$\delta_s = \frac{-g_s^2}{(2\pi)^3 2}(I_{p_2^2} - 2I_{p_2 p_1} + I_{p_1^2})$$

- result depends on the cut ΔE on the energy of a radiated gluon

Gluon bremsstrahlung

- Bremsstrahlung integrals

$$I^{j_1,\dots,j_m}_{i_1,\dots,i_n} = \frac{1}{\pi^2}\int \frac{d^3p_1}{2p_{10}}\frac{d^3p_2}{2p_{20}}\frac{d^3q}{2q_0}\delta(p_0 - p_1 - p_2 - q)\frac{(\pm 2qp_{j_1})\cdots(\pm 2qp_{j_m})}{(\pm 2qp_{i_1})\cdots(\pm 2qp_{i_n})}$$

  p_0 decays into p_1, p_2 and a gluon q

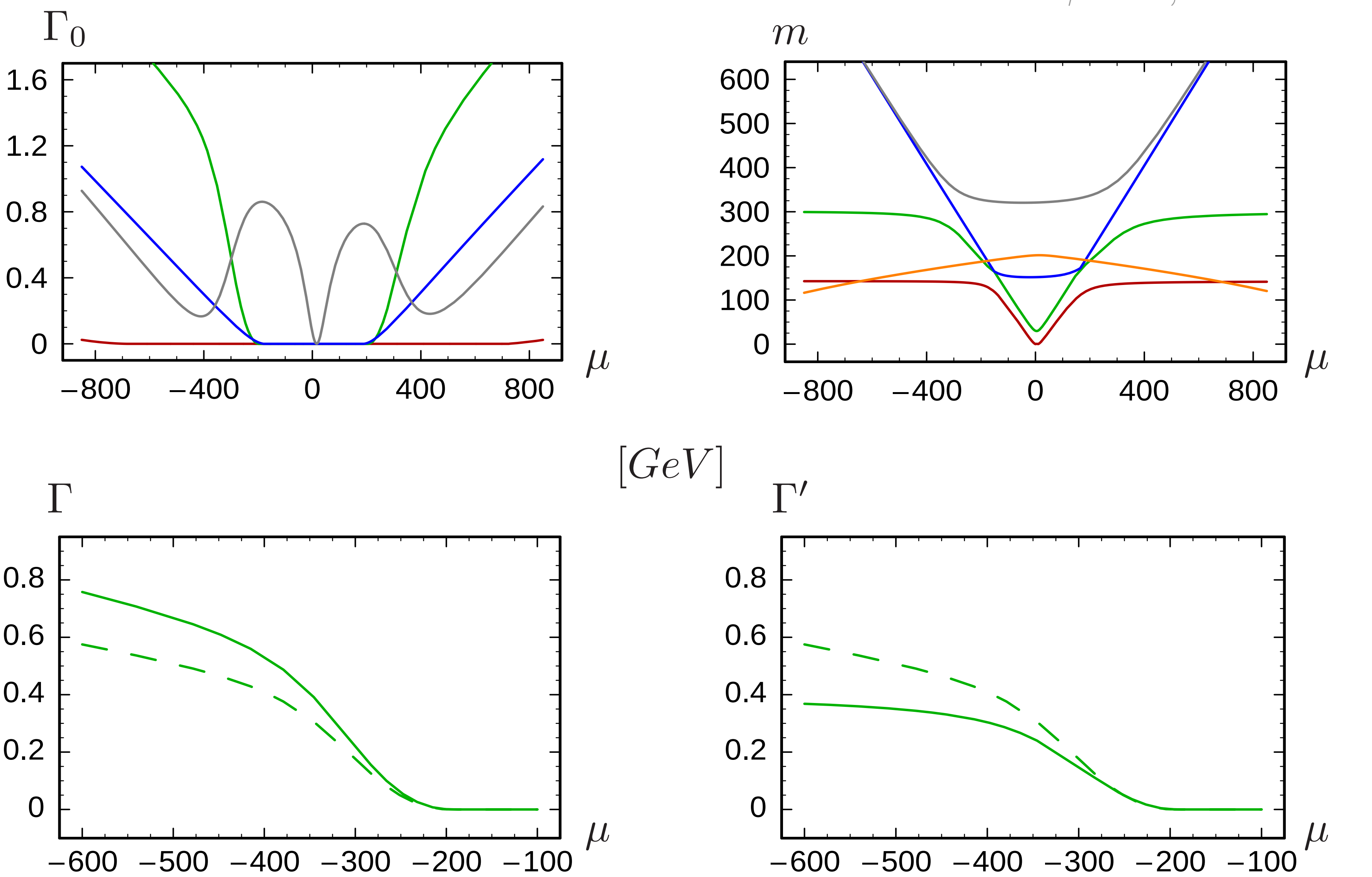
- divergent integrals: $I_{00}, I_{11}, I_{22}, I_{01}, I_{02}, I_{12}$

  taken from: Denner: Techniques for the calculation of electroweak radiative corrections at the one-loop level and results for W-physics at LEP200

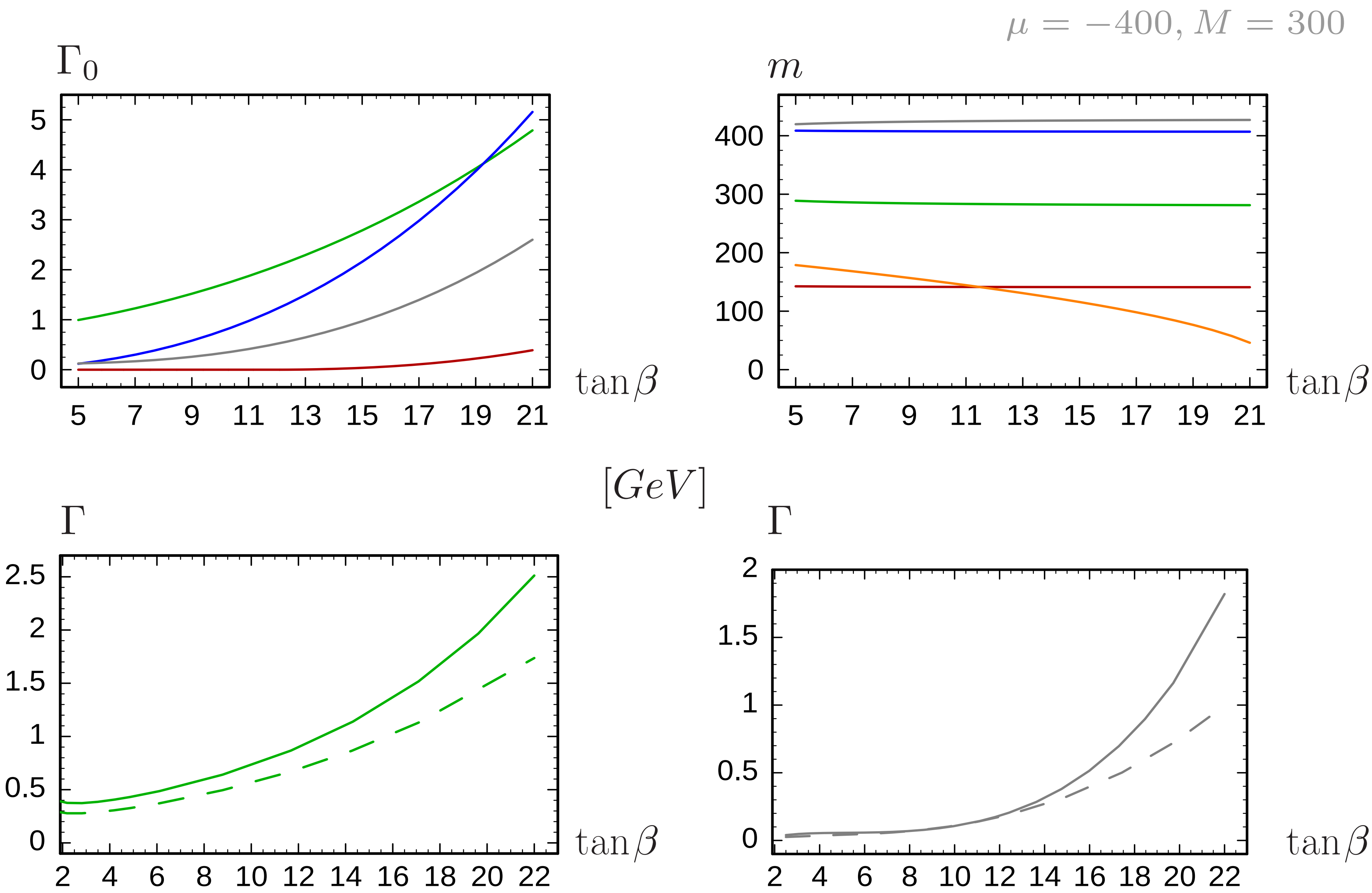
- $\Gamma_{brems} = \frac{1}{2m_0}\frac{g_s^2}{2^5\pi^3}\sum(C^{j_1,\dots,j_m}_{i_1,\dots,i_n} I^{j_1,\dots,j_m}_{i_1,\dots,i_n}) \times$ colour factor

- independent result on the ΔE, integration from 0 to E_{max}

$\tan\beta = 7,\ M = 300$



Numerical results



Soft vs. Bremsstrahlung